

NEUTRINO TRANSPORT AND NEUTRINO LUMINOSITY IN SUPERNOVAE

Inaugural-Dissertation

submitted

for the Degree of Doctor of Philosophy to the
Philosophical Faculty, section II of the
University of Zürich

by

THIERRY JEAN-LOUIS COURVOISIER
from Le Locle NE

Approved by

Prof. Dr. N. Straumann

Zürich 1980

Zentralstelle der Studentenschaft



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I Introduction

During most of the life of a star, neutrinos play only a minor part in the energy balance. This is not so any more during the late stages of the evolution of massive stars ($\sim 10 M_{\odot}$, $M_{\odot}$ is the mass of the sun). The neutrino luminosity of a single star is then comparable or even much larger than the optical luminosity of a whole galaxy.

It is the aim of this thesis to study some aspects of the neutrino physics involved in the most spectacular of the late stages of stellar evolution, namely the catastrophic implosion of the inner core of massive stars. The gravitational energy released by the collapsing core is partly transferred to the outer shells of the star, yielding what is known as a supernova.

The life of a star begins when a dust and gas cloud has shrunk to sufficient densities for thermonuclear reactions to take place. During the longest period of stellar evolution ($\sim 10^8 - 10^9$ years) hydrogen is fused into helium emitting energy mainly as photons and a few neutrinos. The experimental search for these neutrinos from the sun is now underway. At the present time comparatively high energy neutrinos only can be detected by an experiment in which a ^{37}Cl nucleus absorbs a

neutrino and is transformed in ^{37}Ar by the emission of an electron. A further experiment is now being planned in which gallium is used instead of chlorine, with which the main neutrino source in the sun (these neutrinos have low energies), namely $p + p \longrightarrow d + \bar{\nu} + e^+$ will be measured.

When hydrogen is exhausted in the core of a star, the latter contracts further and eventually reaches a density and temperature allowing helium to burn into heavier elements. The cores of massive stars undergo then different stages where carbon, neon, oxygen and silicon are successively burned, finally yielding elements close to ^{56}Fe . ^{56}Fe is the most tightly bound nucleus, it is therefore the end point of the energy emitting thermonuclear reactions. At this stage, the neutrino luminosity has risen to about $10^{15} L_{\odot}$, $L_{\odot}$ is the optical luminosity of the sun, which is some four orders of magnitude more than the optical luminosity of a typical galaxy.

The end of the thermonuclear energy emitting processes brings a further contraction of the core until its temperature is such that a new kind of nuclear reaction sets in: the nuclear photodisintegration. In this reaction, the nuclei are disrupted by the ambient photongas. Another important reaction possible at this stage is the inverse β^- decay : $e^- + p \longrightarrow n + \bar{\nu}$. This latter reaction is allowed when the fermi energy of the electrons is larger

than the energy difference $(m(A, Z) - m(A, Z-1)) \cdot c^2$, where A and Z are the atomic mass and the charge of the nucleus of mass m which absorbs an electron.

Both photodesintegration of nuclei and inverse β -decay lessen the internal pressure sustaining the core against gravitational collapse and lead to its instability. When the internal pressure diminishes, the adiabatical index $\Gamma_1 = (d \ln(p) / d \ln(\rho))_{ad}$ eventually drops below the stability limit $4/3$. What happens then is the dramatic collapse of the inner core of the star to a density of the order of nuclear density or even more.

The possible final stages of stellar evolution are white dwarfs, neutron stars or black holes. Stars of less than $1.4 M_{\odot}$ can settle as white dwarfs, for which the internal pressure is that of a degenerate electron gas. A cold degenerate electron gas cannot support the gravitational attraction of more massive objects than $1.4 M_{\odot}$. These cannot end as white dwarfs but may form neutron stars. This is possible if the mass of the star is less than approximately $3 M_{\odot}$. The internal pressure of a neutron star is that of the degenerate neutron fluid which is its main constituent. This internal pressure is still increased by the repulsive nuclear forces in the neutron fluid. The density of a neutron star is somewhat higher than nuclear density; one may think of a neutron star as a giant nucleus in its ground state.

If the mass of the collapsing star is more than approximately $3 M_{\odot}$, the neutron fluid cannot support the gravitational attraction and a black hole has to form. This means that the collapse does not stop and that an event horizon appears.

It is one of the difficult problems of modern astrophysics to explain how the imploding core of a highly evolved massive star can trigger a supernova explosion and lead to a sufficient mass loss so that a neutron star remains.

That this scenario can happen is known from the Crab nebula and its pulsar. The Crab nebula is an expanding gas shell which resulted from the explosion of a supernova in the year 1054 A.D.. This supernova was observed by chinese astronomers. A pulsar sits in the center of the nebula, which is interpreted as the rapidly rotating neutron star left over from the collapse of the inner core of the exploded star.

Up to the present time no fully convincing quantitative theory of supernova explosions has been provided. It has long been accepted that neutrinos might play a major part in the transmission of energy from the collapsing core to the mantle of the star. Arnett has shown (Arnett 77), however, in 1977 that the neutrinos are trapped in the collapsing core and cannot

exchange enough momentum with matter to reverse the implosion into an explosion. The trapping is due to the elastic scattering of neutrinos on complex nuclei which proceeds via weak neutral currents. The elastic weak interaction of neutrinos on nuclei is coherent for low energy neutrinos, such as those emitted during the collapse of the core. The cross section of the reaction is thus proportional to the square of the atomic weight of the nuclei, and is therefore greatly increased for scattering on complex nuclei. The consequence is that the neutrino diffusion time through the core is larger than the hydrodynamical time characteristic for the collapse; thus the neutrinos are trapped. Neutrinos cannot expel matter because they interact too strongly and not too weakly with matter. This result is confirmed by one of the calculations of this thesis.

Presently, much work is being done to explain supernova explosions by studying the hydrodynamic properties of the collapsing star. One then sees that, depending on the equation of state, the core bounces at a density of $\rho_0 \gg \rho_{\text{nuclear}}$ (Arnett 79, Van Riper 79) and that a reflected shock appears which might eject the outer layer of a star. One of the difficulties of these calculations is to know the equation

of state for matter above nuclear density. This is necessary to predict the properties of the reflected shock and to determine to what extent matter is expelled in a supernova explosion.

A further complexity of supernova theory is the inclusion of rotation and magnetic fields which diminish the symmetry of the problem. This greatly increases the amount of computer time needed to perform the calculation (Müller and Hillebrandt 79).

Most of this thesis deals with the calculation of the transport of neutrinos through the collapsing core of a star. The results allow us to compute the neutrino momentum transfer to the stellar material and to estimate the expected neutrino luminosity of a supernova.

Our study is based on the Boltzmann equation (e.g. Ferziger and Kaper 72). This equation states how the distribution function of a gas changes when collisions between gas particles occur and also in the presence of external forces. In our case the gases are the constituents of the imploding stellar core and the neutrinos. Considering a mixture of two gases denoted 1 and 2, and considering only the collisions between particles of the sort 1 with those of the sort 2, the Boltzmann equation for the gas 1 reads

$$\frac{\partial F_1}{\partial t} + \mathbf{v} \cdot \frac{\partial F_1}{\partial \mathbf{x}} + \mathbf{K} \cdot \frac{\partial F_1}{\partial \mathbf{p}_1} = \quad (1.1)$$

$$\int d^3 p_2 \, d\Omega \, v_{rel} [F_1(p'_1) \cdot F_2(p'_2) - F_1(p_1) \cdot F_2(p_2)] \cdot \frac{d\sigma}{d\Omega},$$

where F_1 (F_2) is the distribution function of the gas 1 (2), p_1 (p_2) is the momentum of the 1 (2) particles and primed quantities refer to the state of the variables after a collision. $\mathbf{K}$ is the exterior force, it vanishes in the following. $d\Omega$ is the angular element and $d\sigma / d\Omega$ is the differential cross section of the interaction considered. The two terms on the right hand side of (1.1) state how many neutrinos enter, resp. leave, the phase space cell $d^3 p_1$ around p_1 under the effect of collisions with particles of the sort 2. An analogous equation to (1.1) gives the change in the distribution function F_2 due to the same collisions 1 with 2. If more collisions (1 with 1, 2 with 2) have to be taken into account, then the corresponding terms appear on the right side of the equations. The left side of equation (1.1), without external force, will be abbreviated by $D_t F_1$.

Most attempts to solve the problem of neutrino transport have used extended computer calculations in order to include as many features of a realistic super-

nova as possible. These methods either integrate a transport equation derived from (1.1) (Arnett 77, 79, Wilson 79, Freedman et al 77) or simulate the neutrino transport (Tubbs 78). The present discussion is different in that it stresses the analytical aspects of the problem. This has the advantage of strongly using the physical properties of the supernova in searching for the solution of the equation (1.1). On the other hand it does not pretend to be very precise, even in a highly simplified model. Nevertheless, the answer to the question explosion or no explosion is clear enough for the conclusion not to depend on details neglected here.

The results presented here are twofold, first the momentum transfer from the neutrinos to the matter is compared to its gravitational attraction. It is shown that the momentum transfer is not more than about 5% of the gravitational attraction which shows that one should not expect the neutrinos (alone) to cause the explosion of the star. Second, we analyse the neutrino luminosity of an imploding core as a function of time, the result of which is presented in figure 5. It gives the luminosity profile for the first few hundredths of seconds following the collapse.

The next section describes the model for which the

calculations have been made, it also briefly summarizes the weak processes needed in the calculations. Section III explains the methods used to solve equation (1.1), whereas the more explicit calculations of the neutrino distribution function is given in section IV. The fifth section is devoted to the comparison of gravitational attraction and momentum transfer. This section relies on the results obtained in section IV. Section VI deals with thermal pair processes in the collapsed core. These processes are not dominant during the collapse but are essential to the calculation of the neutrino luminosity. The neutrino luminosity is then described in section VII.

II The Model

As mentioned in the introduction the aim of this thesis is not to present a fully realistic computer calculation of neutrino transport during stellar collapse. In the present section we describe a simplified model of core collapse. Neutrino transport itself will be discussed in the next sections.

The inner core of the collapsing star is idealized as a homogeneous spherically symmetric mass of iron nuclei (atomic weight 56) of mass $1 M_{\odot}$. The gas of nuclei is described in this model by a Maxwellian distribution function F_n at a temperature T equivalent to 1 MeV (this corresponds to roughly 10^{10} K). The distribution function F_n is further assumed constant in time, leading thus to a linear problem for the neutrino distribution function F_{ν} . The core is further assumed static for the calculation of the neutrino transport. This means that the rotation and the infall velocity are neglected. The density of the core is equal to $2 \times 10^{11} \text{ g cm}^{-3}$ and its radius $R_c = 1.3 \times 10^7 \text{ cm}$. These figures are typical for a collapsing star (Schramm und Arnett 75).

The assumption of a static core will be relaxed in the last sections: An adiabatical free collapse will be considered when we study the electron positron pair

annihilation and the neutrino luminosity. The core will then collapse freely from $\rho \approx 10^{11} \text{ g cm}^{-3}$ to $\rho \gtrsim \rho_{\text{nuclear}}$. It stops at the final density. After the collapse two models are studied: one with the core rebounding to a slightly lower density and one with the core settling without rebound.

It has been shown (Schramm and Arnett 75, Epstein and Arnett 75, Hansen 68) that the main source of neutrinos at $\rho \approx 10^{11} \text{ g cm}^{-3}$ is electron capture by protons. The degeneracy of the electron gas and β -equilibrium require that the energy of the electrons reacting with protons - to yield neutrons and neutrinos - is close to the Fermi energy of the electron gas. This energy is roughly constant during the beginning of the collapse, not far from 24 MeV for $10^{11} \text{ g cm}^{-3} \leq \rho \leq 10^{13} \text{ g cm}^{-3}$ and $T \lesssim 10^{11} \text{ K}$. It follows then that the energy of the emitted neutrinos is also roughly constant at $\bar{E}_\nu \approx 16 \text{ MeV}$. This leads to a narrow initial energy distribution of the neutrino gas. This property of the neutrino distribution function will be used when we calculate the right hand side of the Boltzmann equation. The width Δ of the neutrino distribution function is of order $kT = 1 \text{ MeV}$. A further simplifying idealisation is that the neutrinos are emitted instantaneously at $t = 0$ (cf. section VII for a numerical justification). The

problem we have to solve is thus the transport of a homogeneous flash of neutrinos with energy of approximately 16 MeV in the core of a star consisting of iron nuclei in thermal equilibrium.

The homogeneity of the core requires that the initial neutrino distribution function is also homogeneous throughout the core. As is clear from the left side of the Boltzmann equation, the neutrino distribution function can then depend only weakly on the direction in momentum space. This property will be of great help when we perform phase space integrals.

The rate of elastic neutrino scattering on nuclei is about ten times the neutrino scattering rate on electrons. Elastic scattering on nuclei is therefore the only scattering process considered in the calculation. The importance of the neutrino scattering on nuclei is due to coherence effects (Freedman 74) : The wavelength of a 20 MeV neutrino is larger than the size of the nuclei.

The weak interactions responsible for the elastic scattering of neutrinos on nuclei is described by the Lagrangian

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \ell_\mu \cdot j^\mu + h.c. \quad (2.1)$$

where G_F is the Fermi coupling constant,

$\ell^\mu = \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu$ is the Fierz rearranged neutrino part of the leptonic current and

$$j^\mu = a_0 (V_{I=0}^\mu + \alpha_0 A_{I=0}^\mu) + \quad (2.2) \\ + a_1 (V_{I=1}^\mu + \alpha_1 A_{I=1}^\mu)$$

is the hadronic current (Freedmann et al 77). I denotes isospin, a_i and α_i ($i = 0, 1$) are phenomenological constants.

For elastic scattering on nuclei with $Z = N = A/2$ and zero spin, the only non vanishing contribution comes from the first term of the hadronic current. The correction for nuclei with $Z \neq A/2$ is proportional to $(Z - N) / A$ and will be neglected here. Comparison with the model of Salam and Weinberg for the electro - weak interactions shows that $a_0 = -\sin^2 \theta_w$. We choose the value $a_0^2 = 0.06$, in agreement with the latest laboratory tests.

The elastic scattering cross section for neutrinos on nuclei is easily calculated from the Lagrangian (2.1). Neglecting the recoil of the nuclei, the cross section reads

$$\sigma_{\nu-A} = \frac{G_F^2 a_0^2 A^2}{\pi} E_\nu^2 \quad (2.3)$$

where E_ν is the energy of the neutrino.

The leptonic weak interactions, responsible for the annihilation of electron positron pairs in neutrino anti-neutrino pairs, is described by the phenomenological Lagrangian

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{e} \gamma_\mu (1 - \gamma_5) e \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu + h.c. \quad (2.4)$$

We have neglected the contribution of the neutral weak current, as it is not very important here. From (2.4), one calculates the cross section for pair annihilation:

$$\sigma_{e^+e^-} = \frac{G_F^2}{6\pi E_{e^+} E_{e^-} v_{rel}} \left[m_e^4 + \right. \quad (2.5) \\ \left. + 3 m_e^2 (p_- \cdot p_+) + 2 (p_- \cdot p_+)^2 \right] ,$$

where E_{e^+} (E_{e^-}) is the positron (electron) energy and $p_\pm$ denotes the 4-momenta of the particles.

III Methods

This section gives a short summary of the methods used to find the neutrino distribution function. The more detailed calculations are presented in the next section.

Two different approaches are used to solve the transport equation (1.1). The first approach deals with the neutrino distribution far from thermal equilibrium, while the second describes the neutrino distribution close to thermal equilibrium. The first method relies on the narrowness of the initial distribution function of the neutrino gas as function of the energy, and hence can be used during a short time interval $0 \leq t \leq t_0$ after the neutrino flash. The smallness of the time interval will be specified later. The scattering of neutrinos on nuclei tends to smear the initial distribution function. This trend is also shown by the calculation.

The second method assumes that for $t > t_0$ the neutrino distribution is sufficiently smeared to be reasonably well described by a finite expansion in Laguerre polynomials.

We write the Boltzmann equation (1.1) symbolically in the following form

$$D_t F_\nu = B F_\nu \quad (3.1)$$

where BF_ν stands for the collision term. It is linear in F_ν , since we assume the distribution function of the nuclei to be thermal.

To describe the smearing of F_ν , we introduce a weight function $p(E)$. We choose $p(E) \approx 1$ in the energy interval in which most of the neutrinos are emitted, otherwise we let $p(E) \approx 0$. Figure 1 gives two possible choices for $p(E)$.

We then define

$$F_{in} = F_\nu \cdot p(E) \quad (3.2)$$

$$F_{out} = F_\nu \cdot [1 - p(E)] ,$$

and write equation (3.1) in the form

$$D_t F_\nu = B F_{in} + B F_{out} \quad (3.3)$$

This equation can be approximated by

$$D_t F_\nu = B F_{in} \quad (3.4)$$

for sufficiently small values of t , since $F_{out}(t=0) \ll F_{in}(t=0)$. To solve equation (3.4) we shall make use of the initial distribution function and write

$$F_{in}(E) \approx F_0(\bar{E}_\nu) \cdot p(E).$$

The solution obtained in this way can be used as the first step in the following iteration scheme:

$$10, F_\nu^n = B F_\nu^{n-1}_{in} + B F_\nu^{n-1}_{out}$$

where F_ν^n denotes the n -th iteration step. This method can be used to solve the Boltzmann equation as long as $B F_{out}$ may be considered as a perturbation. Another

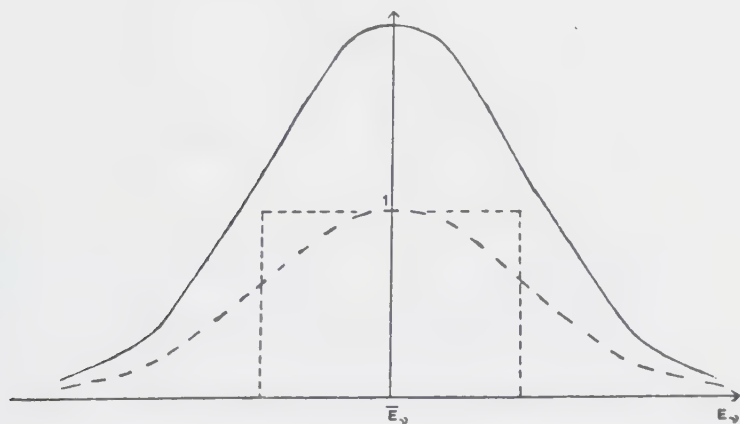


Fig 1. The initial neutrino distribution (solid line) is given together with two possible choices for the weight function $p(E)$ (dashed lines).

method has to be used when the neutrino distribution function is significantly smeared. This gives the defining criterion for t_0 . The main advantage of the above method is that it approximates the distribution function best where it is largest and thus approximates the integral BF_ν reasonably well.

For $t > t_0$, we expect the neutrino gas to be closer to thermal equilibrium. Since the phase space occupied by the emitted neutrinos is small compared to the total available phase space (neutrinos are emitted with a well defined energy) the equilibrium distribution function is $\exp(-E_\nu/kT)$. A good expansion for F_ν is then an expansion in Laguerre polynomials. F_ν has the form

$$F_\nu = e^{-E_\nu/kT} \cdot \sum_{n=0}^{\infty} c_n(t, r, \mu) \cdot L_n(E_\nu) \quad (3.5)$$

where L_n are the laguerre polynomials and c_n are coefficients depending on t , r and μ , where $\mu = \cos \angle_{\vec{p}, \vec{r}}$. c_n are to be determined by the transport equation and the boundary and initial conditions. The Boltzmann equation reduces to the following equation for the coefficients c_n

$$ID_t c_m = \sum_n (M_{mn} \cdot c_n) \quad (3.6)$$

where M_{mn} is the $(k + 1)$ - dimensional projection of the Boltzmann operator on a functional space spanned by the $(k + 1)$ first laguerre polynomials. The solution of equation (3.6) is easily expressed in terms of the eigenvalues λ_i and eigenvectors $\underline{v}_i$ of M_{mn} :

$$\underline{c} = \sum_i^{\kappa} p_i e^{\lambda_i t} \underline{v}_i \quad (3.7)$$

where $\underline{c}$ is the vector $\begin{pmatrix} c_0 \\ \vdots \\ c_k \end{pmatrix}$ and p_i are factors determined by the initial conditions.

IV Neutrino Transport

The methods sketched in the previous section are developed here in greater detail, and the calculations leading to the neutrino distribution function are presented. The first step of this development is common to both cases $t < t_0$ and $t > t_0$, leading to a slightly simplified form of equation (1.1); this will be discussed first. We then treat the two cases $t < t_0$ and $t > t_0$ separately.

We first transform the collision integral into a more practical form (Cercignani 75). Neglecting the energy and the recoil of the nucleus in the scattering cross section, we can write the second term of the collision integral (the second term of the right hand side of equation (1.1) with $1 \equiv \nu$, $2 \equiv n$) in the form

$$f(E_\nu) \cdot F_\nu(p_\nu) \quad (4.1)$$

where

$$f(E_\nu) = \int d^3p_n F_n(p_n) \cdot \sigma = n \cdot \sigma, \quad (4.2)$$

and $f(E_\nu)$ is the collision frequency, n is the number density of the nuclei and σ is the total cross section

for elastic scattering of neutrinos on nuclei (see equation (2.3)). The first term in the collision integral can be written as

$$\int \frac{d^3 p'_v}{2 p'_{v0}} F_v(p'_v) \cdot K(p_v, p'_v) \quad (4.3)$$

where

$$K(p_v, p'_v) = (2\pi)^0 \int d^3 p_n F_n(p_v + p_n - p'_n) \cdot \quad (4.4)$$

$$\cdot \delta(p'_{v0} + p'_{n0} - p_{v0} - p_{n0}) \cdot \frac{|T|^2}{2 p_{v0} \cdot 2 p_{n0}}$$

T is the T - Matrix element for the reaction, defined by the equation

$$\begin{aligned} \langle p'_v p'_n | S - 1 | p_v p_n \rangle &= \\ &= -i (2\pi)^4 \cdot \delta^4(p'_v + p'_n - p_v - p_n) \langle p'_v p'_n | T | p_v p_n \rangle. \end{aligned}$$

The normalisation of the single particle states is

$\langle p | p' \rangle = 2 p_0 \delta^3(p - p')$. To obtain (4.3) from (1.1) one first writes the neutrino transition rate in the interval $d^3 p_v$ about p_v as follows:

$$(2\pi)^6 \int F_v(p'_v) \frac{d^3 p'_v}{2 p'_{v0}} \cdot F_n(p'_n) \frac{d^3 p'_n}{2 p'_{n0}} \cdot dW(p'_v p'_n \rightarrow p_v p_n), \quad (4.5)$$

where

$$dw(p_v p_n \rightarrow p'_v p'_n) = |\langle p'_v p'_n | S - 1 | p_v p_n \rangle|^2 \frac{d^3 p'_v}{2 p'_{v0}} \frac{d^3 p'_n}{2 p'_{n0}}$$

is the transition probability. The integral in (4.5) does not extend over p_v . We use momentum conservation to perform the integral in (4.5) which leads immediately to equation (4.3.). Using (4.1.) and (4.3) we write the Boltzmann equation in the form

$$\begin{aligned} D_t F_v(p_v) = & \int \frac{d^3 p'_v}{2 p'_{v0}} F_v(p'_v) \cdot K(p_v, p'_v) - \\ & - f(E_v) \cdot F_v(p_v) \end{aligned} \quad (4.6)$$

The distribution function of the nuclei is thermal, i.e.

$$F_n(p) = C(T) e^{-p^2/2m_n T} \quad , \quad (4.7)$$

where $C(T)$ is given by the normalisation condition

$$n = \int d^3 p F_n(p) \quad ,$$

thus

$$C(T) = \frac{n}{2\pi \Gamma(\frac{3}{2}) (2m_n T)^{3/2}} \quad , \quad (4.8)$$

where m is the mass of the nuclei.

The explicit form of the T - matrix element using the Lagrangian (2.1) is

$$|T|^2 = 16 \frac{G_F^2 a_0^2 A^2}{(2\pi)^{12}} \cdot m^2 E_\nu^2 (1 + \cos\theta) , \quad (4.9)$$

where θ is the angle between $\underline{p}_\nu$ and $\underline{p}'_\nu$. The large mass of the nuclei implies that their rest reference frame is to a good approximation the center of mass reference frame for the neutrino scattering. Furthermore, the small kinetic energy of the nuclei ($\sim kT$) implies that they are almost at rest in the reference frame of the star. Thus the core rest frame coincides approximately with the center of mass reference frame for neutrino scattering.

The Kernel $K(p_\nu, p'_\nu)$ can be explicitly calculated using equations (4.7) and (4.9). The approximate equivalence of the center of mass system and the star rest frame is used to perform the integration over d^3p_n , and only the leading term in E_ν/m is retained in the result. This leads to

$$K(p_\nu, p'_\nu) = D(T) \cdot E_\nu \frac{(1 + \cos\theta)(E'_\nu - E_\nu)^2}{|E'_\nu \cos\theta - E_\nu|^3} \cdot \quad (4.10)$$

$$\cdot \exp - \left[m (E'_\nu - E_\nu)^2 / 2 k T (E'_\nu \cos\theta - E_\nu)^2 \right] .$$

The factor $D(T)$ is

$$D(T) = \frac{m^3}{\pi} \cdot C(T) \cdot G_F^2 \alpha_0^2 A^2 \quad . \quad (4.11)$$

The assumed isotropy of the neutrino distribution function in momentum space is now used to perform the angle integration in (4.3). This gives the following expression for the first term of the collision integral:

$$\begin{aligned} & \int \frac{d^3 p'_\nu}{p'_{\nu 0}} F_\nu(p'_\nu) \cdot K(p_\nu, p'_\nu) = \\ & = D(T) \pi E_\nu \int E'_\nu dE'_\nu F_\nu(E'_\nu) \cdot \left\{ \frac{kT}{m} \frac{E'_\nu + E_\nu}{E'^2_\nu} \cdot \right. \\ & \cdot \exp - \left[\frac{m}{2kT} \frac{(E'_\nu - E_\nu)^2}{(E'_\nu + E_\nu)^2} \right] + \left(\frac{\pi kT}{2m} \right)^{1/2} \frac{E'_\nu - E_\nu}{E'^2_\nu} \cdot \\ & \cdot \left[\operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \cdot \frac{E'_\nu - E_\nu}{E'_\nu + E_\nu} \right) \mp 1 \right] \left. \right\} \quad . \end{aligned} \quad (4.12)$$

The - sign is to be used when $E_\nu < E'_\nu$; the + sign when $E_\nu > E'_\nu$. The error function erf is defined by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad .$$

We now have to treat the two cases $t < t_0$ and $t > t_0$ separately. For $t < t_0$ the neutrino distribution function is approximated by

$$F_{in}(E'_\nu) = F_\nu(\bar{E}_\nu) \cdot p(E'_\nu) \quad (4.13)$$

as discussed in section III. $\bar{E}_\nu$, the mean energy of the emitted neutrinos, is 16 MeV. We choose for the weight function $p(E)$ a gaussian function:

$$p(E_\nu) = e^{-(\bar{E}_\nu - E_\nu)^2 / \Delta^2}, \quad (4.14)$$

where Δ is the width of the initial neutrino distribution function.

It is a good approximation to write $E'_\nu + E_\nu = 2\bar{E}_\nu$ in equation (4.12), because the function $p(E'_\nu)$ essentially vanishes for $|E'_\nu - \bar{E}_\nu| \gtrsim \Delta$, and because the expression between the brackets $\{\}$ converges rapidly to zero for $|E'_\nu - E_\nu| \gtrsim \Delta_E$, where Δ_E^2 is the quantity

$$\Delta_E^2 = \frac{8kT\bar{E}_\nu^2}{m}.$$

It is straightforward to integrate the first term between the brackets $\{\}$ in equation (4.12). To compute the integral containing the expression in squared brackets in (4.12) one notes that $\Delta_E \ll \Delta$, and therefore

$$e^{-(E'_\nu - \bar{E}_\nu)^2 / \Delta^2} \approx e^{-(E_\nu - \bar{E}_\nu)^2 / \Delta^2}.$$

One obtains finally

$$\int \frac{d^3 p'_\nu}{2 p'_{\nu 0}} F_\nu(p'_\nu) K(p_\nu, p'_\nu) = \quad (4.15)$$

$$= D(T) \prod F_\nu(\bar{E}_\nu) \cdot \left\{ 2 \bar{E}_\nu \frac{kT}{m} \left(\frac{\pi \Delta^2 \Delta_E^2}{\Delta_E^2 + \Delta^2} \right)^{1/2} e^{-\frac{(E_\nu - \bar{E}_\nu)^2}{\Delta_E^2 + \Delta^2}} - \right.$$

$$\left. - \left(\frac{\pi kT}{2 m} \right)^{1/2} \frac{\Delta_E^2}{2} \exp - \left[(E_\nu - \bar{E}_\nu)^2 / \Delta^2 \right] \right\}$$

We abbreviate this as $s(E_\nu) \cdot F_\nu(\bar{E}_\nu)$ thus defining $s(E_\nu)$.

With this result, the Boltzmann equation has the form:

$$D_t F_\nu = s(E_\nu) F_\nu(\bar{E}_\nu) - f(E_\nu) \cdot F_\nu(E_\nu) . \quad (4.16)$$

The solution of (4.16) is easily found to be

$$F_\nu = s(E_\nu) \cdot \gamma \cdot \frac{\alpha(t, r, \mu)}{\bar{s} - f + p(E_\nu) \cdot f(E_\nu)} \cdot \quad (4.17)$$

$$\cdot \left[e^{-(\bar{s} - f)t} - e^{-f(E_\nu)p(E_\nu)t} \right] +$$

$$+ \gamma \cdot \alpha(t, r, \mu) \cdot p(E_\nu) \cdot e^{-f(E_\nu)p(E_\nu)t} ,$$

where we have introduced the notations $f = f(\bar{E}_\nu)$ and $\bar{s} = s(\bar{E}_\nu)$. f is the scattering frequency of a typical neutrino. In our model it has the numerical value

$f = 4.2 \times 10^4 \text{ s}^{-1}$. If one takes $\Delta = kT = 1\text{MeV}$, then

the numerical value of $\bar{s}$ is $3.7 \times 10^4 \text{ s}^{-1}$. $t_s = \bar{s}^{-1}$ is the typical time interval between two collisions. Numerically, $t_s = 2.4 \times 10^{-5} \text{ s}$, and the mean free path $\Lambda = c \cdot t_s = 7.1 \times 10^5 \text{ cm}$. The constant γ is the density of neutrino in phase space at $t = 0$, its possible numerical value will be discussed in the next sections.

The function α is determined by the following requirements: i) the neutrino flash at $t = 0$ is homogeneous in the core, ii) during the first few t_s the neutrinos escape almost freely from the core and iii) $\partial_t \alpha = 0$. These requirements lead to

$$\alpha = \Theta(R - |r - r_c|), \quad (4.18)$$

where

$$\Theta(x) = \begin{cases} 1 & , x > 0 \\ 0 & , x < 0 \end{cases}.$$

As mentioned in section III the solution given by (4.17) and (4.18) of the transport equation is a reasonably good approximation to the real distribution function of the neutrino gas only for a few scattering times. The neutrino distribution function doubles its width when its central value is down by a factor 2 from its initial value. A reliable evaluation of the

validity time t_0 is therefore the decay time of $F_\nu(\bar{E}_\nu)$, which is $15 - 41^{-1}$. This gives

$$t_0 = 15 - 41^{-1} = 2 \times 10^{-4} \text{ s} . \quad (4.19)$$

The distribution function (4.17) is only a poor approximation to the solution of the Boltzmann equation for $t \sim t_0$, because (4.17) completely neglects BF_{out} . It is however possible to make a better approximation, which takes BF_{out} into account, by using the iteration procedure proposed in section III.

For $t > t_0$ the energy dependence of the neutrino distribution is approximated by a finite number of Laguerre polynomials. We write the Boltzmann equation, using (4.12), in the form

$$\begin{aligned} 10_+ F_\nu &= \int dE'_\nu \, K(E_\nu, E'_\nu) F_\nu(E'_\nu) - \\ &- f(E_\nu) \cdot F_\nu(E_\nu) , \end{aligned} \quad (4.20)$$

where

$$\begin{aligned} K(E_\nu, E'_\nu) &= O(T) \pi E_\nu E'_\nu \left\{ \frac{kT}{m} \frac{E'_\nu + E_\nu}{E_\nu^2} \exp - \left[\frac{m}{2kT} \frac{(E'_\nu - E_\nu)^2}{(E'_\nu + E_\nu)^2} \right] + \right. \\ &+ \left. \left(\frac{\pi kT}{2m} \right)^{1/2} \frac{E'_\nu - E_\nu}{E_\nu^2} \left[\operatorname{erf} \left(\sqrt{\frac{m}{2kT}} \cdot \frac{E'_\nu - E_\nu}{E'_\nu + E_\nu} \right) + 1 \right] \right\} . \end{aligned}$$

The same symbol K is used here as in (4.12) for the integral kernel, although the angle integration is performed here. No confusion is possible with K as it was used before.

The neutrino distribution function is of the form given in (3.5):

$$F_\nu = e^{-E_\nu/kT} \cdot \sum_{n=0}^{\infty} c_n(t, r, \mu) L_n(E_\nu/kT) . \quad (4.21)$$

We multiply (4.21) with L_m and integrate over the energy. Using the orthogonality relations

$$\int dx e^{-x} L_m(x) L_n(x) = \delta_{mn} , \quad (4.22)$$

we obtain the following equation for the $c_m(t, r, \mu)$:

$$\partial_t c_m = kT \int dx' dx [L_m(x) K(x, x') e^{-x'} \cdot \quad (4.23)$$

$$\cdot \sum_{n=0}^{\infty} c_n L_n(x')] - \int dx L_m(x) e^{-x} f(x) \sum_{n=0}^{\infty} c_n L_n(x) .$$

The dimensionless variables $x = E_\nu/kT$ and $x' = E_\nu'/kT$ have been introduced in (4.23). We now transform the variables r, t in $v = t$ and $u = t - r/\mu$, so that

$$\partial_t = \frac{\partial}{\partial v} . \text{ Introducing the matrices}$$

$$Q_{mn} = \kappa_T \int dx' dx L_m(x) \kappa(x, x') e^{-x'} L_n(x') \quad (4.24)$$

and

$$\omega_{mn} = \int dx L_m(x) e^{-x} f(x) L_n(x) ,$$

we can write (4.23) in the concise form

$$\frac{J c_m}{J_v} = \sum_n (Q_{mn} - \omega_{mn}) c_n . \quad (4.25)$$

The solutions of this equation are of the form

$$c(u, v) = \sum_{i=0}^{\infty} p_i(u) e^{\lambda_i v} \underline{v}_i . \quad (4.26)$$

$\underline{c}(u, v)$ is the vector $\begin{pmatrix} c_0 \\ \vdots \\ c_k \end{pmatrix}$; λ_i are

the eigenvalues and $\underline{v}_i$ the eigenvectors of the matrix $Q_{mn} - \omega_{mn}$. The $p_i(u)$ are determined from the initial conditions.

It follows from the orthogonality relations (4.22) that

$$\int dx F_v(t=t_0) L_m(x) = c_m(t=t_0) . \quad (4.27)$$

Using (4.26) in (4.27), we obtain the relation

$$e^{\lambda_i t_0} p_i(t=t_0) = \sum_n (V_{in})^{-1} c_n(t=t_0) ,$$

where V_{in} is the n^{th} component of the i^{th} eigenvector of $Q_{mn} = \omega_{mn}$. As initial condition we choose the function

$$F_\nu(t=t_0) = \gamma \beta(u) \Big|_{t=t_0} \cdot \exp - \left[\frac{(E_\nu - \bar{E}_\nu)^2}{(2\Delta)^2} \right] , \quad (4.28)$$

which has the same general energy dependence as the result of the previous calculation of F_ν for times $t < t_0$. γ and Δ are the same constants as previously, so that the width of the distribution function at $t = t_0$ is twice the initial width, in agreement with the definition of t_0 . The use of γ (instead of e.g. $\gamma/2$) for the neutrino spectral density at $t = t_0$ assumes further neutrinoisation of the matter, thus filling phase space at $E_\nu = \bar{E}_\nu$ again. (Further discussions of the numerical value of γ are given in the next sections). The function $\beta(u)$ will be specified when we study the boundary conditions.

One sees from equations (4.26), (4.27) and (4.28) that

$$p_i(u) = p_{i0} \cdot \gamma \cdot \beta(u) \quad (4.29)$$

where $\rho_{i0} = \rho_i(t = t_0)$.

The matrix Q_{mn} was calculated numerically for $k = 5$ and $\Delta = 1\text{MeV}$. Table 1 gives the eigenvalues of $Q_{mn} - \omega_{mn}$, λ_i , and the dimensionless coefficients $\rho_{i0} \cdot v_{in}$, which multiplies $e^{\lambda_i t}$ in c_n . The use of only 6 Laguerre polynomials is justified by the modest precision we need. The same calculation was performed using 5 polynomials and the result described in the next section did not differ by more than 30%. As will be seen then, an uncertainty of 30% does not affect the conclusions at all.

The function $\beta(u)$ is now calculated using the boundary condition: The neutrinos stream freely out of the core, once they have reached its surface. Writing the neutrino distribution for $t > t_0$ in the form

$$F_\nu = \gamma \cdot \beta(u) \quad \gamma_2(E_\nu, t) \quad , \quad (4.30)$$

which defines $\gamma_2(E_\nu, t)$ implicitly, we remark that, since neither absorption nor emission processes are included in the collision operator, the integral

$\int d^3 p_\nu \gamma_2 =: n_0$ is a constant. The neutrino density n_ν is $n_\nu = \int d^3 p_\nu F_\nu(p_\nu) = \gamma \beta n_0$ and therefore

$$\dot{n}_\nu = \gamma n_0 \dot{\beta} \quad . \quad (4.31)$$

Table 1. The eigenvalues λ_i and the coefficients $\rho_{oi} \cdot v_{in}$ used in the text are given for each i resp. for each pair (i, n) .

i	0	1	2	3	4	5	
λ_i	0.06	-0.24	-1.4	-4.8	-14	-14 ²	$\cdot 10^{25} \cdot v_{in}^{-1}$
n							
0	12	-30	54	-33	0	0	
1	11	5.7	-130	182	-125	0	
2	8.8	19	9.1	-300	540	69	
3	5.7	17	85	56	-970	-416	
4	2.5	9.4	74	280	680	1500	
5	0.59	2.5	24	113	460	3100	

In order to estimate $\dot{\beta}$ we first consider the equation of continuity for the neutrino gas. Considering the volume V between two concentric spheres of radius r and $r - \epsilon$, the equation of continuity reads

$$\begin{aligned} \frac{d}{dt} \int_V n(r) d^3x = & - \int_r d\Omega \langle v^r \rangle \cdot n(r) + \\ & + \int_{r-\epsilon} d\Omega \langle v^r \rangle \cdot n(r) , \end{aligned} \quad (4.32)$$

where $\langle v^r \rangle$ is the mean radial velocity of the neutrino gas, positive when directed towards the exterior of the core. (This quantity will be estimated at the core surface below.) The comparison of the coefficients of ε in (4.32) leads to the equation

$$\dot{n}(r) = \frac{-2 \cdot n \cdot \langle v^r \rangle}{r} . \quad (4.33)$$

When inserted into equation (4.31), (4.33) gives an expression for $\dot{\beta}$. The rate at which the neutrinos leave the surface of the core is given by the mean value $\bar{\mu}$ of their radial velocity throughout the core. $\bar{\mu}$ is roughly estimated by noting that the neutrinos in the shell $R-\Delta \leq r \leq R$ have a mean μ of $1/2$, since no neutrino comes from outside the core, whereas the neutrinos inside $r = R - \Delta$ have a much lower mean radial velocity. Therefore $\bar{\mu}$ is approximately

$$\bar{\mu} = \frac{\frac{1}{2} \cdot N_b \text{ of } \nu \text{ with } \mu \sim \frac{1}{2}}{\text{total } N_b \text{ of } \nu} . \quad (4.34)$$

Using the homogeneity assumption, the ratio of the number of neutrinos with $\mu \sim 1/2$ to the total number of neutrinos is equal to the ratio of the volume of the shell $R-\Delta < r < R$ to the volume of the core.

To first order in Λ/R one finds

$$\bar{\mu} = \frac{3}{2} \Lambda / R \quad (4.35)$$

Using this value for $\langle v^r \rangle$ in (4.33), gives

$$\beta(R, t) = e^{-\frac{3\Lambda}{R^2}(t-t_0)} \quad (4.36)$$

The constant appearing in (4.36) ensures that the spectral density of neutrinos does not exceed γ at $t = t_0$, as demanded by the initial conditions.

This result terminates our discussion of the neutrino distribution function. The next section uses the information contained in F_ν to make predictions about the collapse of the core.

V Neutrino Pressure vs Gravitation

The neutrino distribution functions, computed in the previous section, are now used to evaluate the relative importance of momentum transfer from the neutrino gas to the nuclei in the core, as compared to the gravitational attraction of the core. We study the relative effects of gravitation and neutrino transport by considering the equation of motion for the core. The equation of motion in general is contained in

$$T^{\mu\nu}{}_{;\nu} = 0 \quad , \quad (5.1)$$

where $T^{\mu\nu}$ is the stress energy tensor. In the present discussion $T^{\mu\nu}$ has two parts:

$$T^{\mu\nu} = T^{\mu\nu}_M + T^{\mu\nu}_\nu . \quad (5.2)$$

$T^{\mu\nu}_M$ is the stress energy tensor of nuclei and $T^{\mu\nu}_\nu = \int \frac{d^3 p_\nu}{p_{\nu 0}} p^\mu_\nu p^\nu_\nu F_\nu(p)$, that of the neutrino gas.

The r component of equation (5.1) for weak gravitational fields, neglecting the internal pressure of the nuclei gas, reads

$$g^{ir} = - \frac{G \rho M_r}{r^2} - T^{r0}{}_{,0} - T^{rr}{}_{,r} . \quad (5.3)$$

Here we have used $\frac{M}{T^{ro}} = g \cdot v^r$. M_r is the mass inside a sphere of radius r .

The components of $\dot{T}^{\mu\nu}$ appearing in equation (5.3) are

$$\dot{T}^{ro} = \int E_v^3 dE_v d\Omega \mu F_v, \quad (5.4)$$

and

$$\dot{T}^{rr} = \int E_v^3 dE_v d\Omega \mu^2 F_v.$$

The particular form in which the derivatives of the distribution function appear in the equation of motion and the vanishing of $D_t \alpha$ and $D_t \beta$ (remember that α and β are functions of u) lead to the following expression for $\dot{v}^r$:

$$\dot{v}^r = -\frac{G M_r}{r^2} - \frac{1}{\rho} \int E_v^3 dE_v d\Omega \mu \cdot \left\{ \begin{array}{l} \gamma \propto \dot{\psi}_1 \\ \gamma \beta \dot{\psi}_2 \end{array} \right\}, \quad (5.5)$$

where the index 1 indicates $t < t_0$ and the index 2 indicates $t > t_0$. The notation of (4.30) for $F_v(t > t_0)$ has been extended to $F_v(t < t_0)$ in an obvious way (compare with equation (4.17)). We first consider the integral

$$a_{\nu}^{(1)} = -\frac{1}{\rho} \int E_{\nu}^3 dE_{\nu} d\Omega \mu \gamma \propto \dot{\phi}_1 ,$$

which appears in (5.5). In order to calculate the integral over $d\Omega$ we have to consider different regions of space time as described in figure 2.

The angular integral vanishes in region 1 expressing the fact that the net flux of neutrinos is zero in this region of space time. In region 2, one has

$$\gamma \int \mu \propto d\Omega = \pi \gamma \left[1 - \frac{(r-R)^2}{t^2} \right] . \quad (5.6)$$

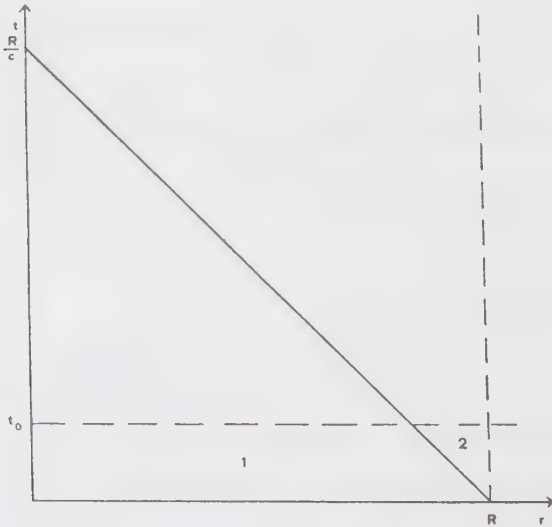


Fig. 2: The two regions of space-time distinguished in the calculation of $a_{\nu}^{(1)}$ are shown. The net flux of neutrinos vanishes in region 1.

In the following, we restrict the calculations to the study of the equation of motion at $t = 0$, resp. at $t = t_0$. It is then possible to perform all the calculations analytically. Since the functions are smooth in the time variable and the results unequivocal, the conclusions are expected to hold for any time.

From equation (4.17) one sees that

$$\dot{\psi}_1(t=0) = s - \frac{1}{2} p^2 \quad (5.7)$$

which, when integrated over the energy, gives the following contribution to $a_v^{(1)}$:

$$\int E_v^3 dE_v \dot{\psi}_1 \Big|_{t=0} = \sqrt{\pi} \bar{E}_v^3 \Delta \cdot \left[\frac{3}{2} \left(1 + \frac{3}{2} \left(\frac{\Delta}{\bar{E}_v} \right)^2 \right) - \frac{1}{2} \left(1 + 30 \left(\frac{\Delta}{\bar{E}_v} \right)^2 \right) \right] , \quad (5.8)$$

where we have made use of the approximation $(\Delta^2 + \Delta_E^2)^{1/2} \approx \Delta$ and neglected the terms of order $(\Delta / \bar{E}_v)^4$. Inserting this result in (5.5) leads to

$$a_v^{(1)}(r=R, t=0) = -\frac{\pi \gamma}{\rho} \sqrt{\pi} \bar{E}_v^3 \Delta \cdot \left[\frac{3}{2} \left(1 + \frac{3}{2} \left(\frac{\Delta}{\bar{E}_v} \right)^2 \right) - \frac{1}{2} \left(1 + 30 \left(\frac{\Delta}{\bar{E}_v} \right)^2 \right) \right] . \quad (5.9)$$

We now calculate the analogous expression for $t = t_0$:

$$a_{\nu}^{(2)}(t_0, R) = -\frac{1}{\rho} \gamma \beta \int d\Omega dE_{\nu} \mu E_{\nu}^3 \dot{\phi}_2 \Big|_{t_0, R}.$$

The contribution of the angle integral to a $\dot{\phi}_2^{(2)}$ is

$4\pi \cdot \langle v^r \rangle(R) = 4\pi \bar{\mu}$, since $\bar{\mu}$ is the mean velocity with which the neutrinos reach the surface of the core.

Next, we use the representation

$$x^3 = 6 L_0(x) - 18 L_1(x) + 18 L_2(x) - 6 L_3(x)$$

of x^3 in terms of Laguerre polynomials, and the orthogonality relation (4.22) to get

$$a_{\nu}^{(2)}(R, t_0) = -\frac{1}{\rho} 4\pi \bar{\mu} \gamma (\kappa T)^4. \quad (5.10)$$

$$\cdot [6 \dot{c}_0 - 18 \dot{c}_1 + 18 \dot{c}_2 - 6 \dot{c}_3]_{t=t_0}.$$

The numerical value of the expression in the squared brackets is calculated from table 1.

The last information needed to complete the discussion of the equation of motion is the possible range of values for the spectral density γ . We first remark that the density of neutrinos in phase space (hence the distribution function) is limited by the exclusion principle of

Pauli: The number of neutrinos dN in a phase space cell $d^3p d^3x$ is $dN \leq 1/h^3 \cdot d^3p d^3x$, since neutrinos have only one helicity state. This means, in our normalisation conditions

$$F_\nu \leq 1/h^3 \quad \text{or} \quad \gamma \leq 5.2 \times 10^{29} \text{ MeV}^{-3} \text{ cm}^{-3}. \quad (5.11)$$

We use the highest value of γ , because it maximises the effect of neutrino transport vs gravitation. The true values of a_ν can then only be lower than those given here. We are now in position to give the numerical value of a_ν for the model discussed in section II:

$$a_\nu^{(1)}(t=0, \rho = 2 \times 10^{11} \text{ g cm}^{-3}) = 3.2 \times 10^{10} \text{ cm s}^{-2} \quad (5.12)$$

and

$$a_\nu^{(2)}(t=t_0, \rho = 2 \times 10^{11} \text{ g cm}^{-3}) = 1.7 \times 10^{10} \text{ cm s}^{-2}. \quad (5.13)$$

A similar calculation to the one presented here, to obtain (5.13), but using 5 Laguerre polynomials instead of 6 has given $a_\nu^{(2)}(t = t_0) = 1.2 \times 10^{10} \text{ cm/s}^2$. This differs by less than 30% from (5.13). The compatibility of the two numbers gives some confidence in the method we have used.

The gravitational attraction at the surface of the core, in the Newtonian approximation, is

$$a = -GM/R^2 = 8 \times 10^{11} \text{ cm s}^{-2}, \quad (5.14)$$

for the model in discussion.

The comparison between (5.12) and (5.13) on one side and (5.14) on the other side shows that the outward acceleration induced by the neutrinos a_v is only about 5% of the gravitational acceleration g. It is therefore unreasonable to expect that neutrino transport is the leading mechanism of supernova explosions. This conclusion corroborates the recent numerical work on this subject (see e.g. Arnett 79).

A further information that can be gained from the neutrino distribution function is the time profile of the neutrino pulse as it might be observed (Schramm 79). This will be discussed in section VII, although for a somewhat simplified description of the transport as the one used in this section.

VI Electron Positron Pair Annihilation

In this and in the next sections, we consider, instead of a static core, one that is freely falling. This is consistent with the result of the last section where it was shown that the neutrino pressure may be neglected. The free fall is stopped when the core has reached a density slightly over nuclear density. We then consider two models, a simple bounce to a density slightly below nuclear density and a model where the core settles without bouncing at a density slightly above nuclear density. The core is assumed to collapse adiabatically, which is a good approximation, because the neutrinos which are trapped, cannot carry much energy out of the core during the collapse.

Electron - positron pair annihilation into a neutrino-antineutrino pair is considered in this section. The corresponding neutrino luminosity is discussed for the same model in the next section.

The total cross section for the electron -positron pair annihilation in the $V - A$ phenomenological theory was given in equation (2.5), we recall it for convenience:

$$\sigma_{e^+e^-} = \frac{G_F^2}{6 \pi E_- E_+ v_{rel}} \left[m_e^4 + 3 m_e^2 (P_- \cdot P_+) + (6.1) \right. \\ \left. + 2 (P_- \cdot P_+)^2 \right] ,$$

E_- and E_+ are the energies of the electron, resp. of the positron, and p_+ and p_- are their respective 4-momenta.

In the extreme relativistic case one finds the mean angular value of the cross section to be

$$\langle \sigma \cdot v \rangle = \frac{8}{3} \frac{G_F^2}{6\pi} E_+ E_- . \quad (6.2)$$

This expression has to be integrated over the initial distributions of the electrons and positrons, in order to calculate the neutrino production rate per unit volume λ . This integral is first performed without considering the Pauli inhibition factor for the final neutrino states. The necessary correction will be discussed later in this section. λ is then

$$\lambda = \int F(e^-) \cdot F(e^+) \cdot \langle \sigma \cdot v \rangle d^3p_+ d^3p_- , \quad (6.3)$$

where $F(e^-)$ is the distribution function of the electron gas, $F(e^+)$ that of the positron. For a strongly degenerate electron gas the distribution function is approximately

$$F(e^-) = \begin{cases} \frac{2}{(2\pi)^3} & , E_- < E_F^{e^-} \\ 0 & , E_- > E_F^{e^-} \end{cases} , \quad (6.4)$$

E_F^- is the Fermi energy of the electron gas. For electrons and positrons in thermal equilibrium with the photon gas we have the condition $E_F^{e^-} + E_F^{e^+} = 0$, because the chemical potential of the photons vanishes. When the electron gas is strongly degenerate, the positron gas is strongly non degenerate and its distribution function is approximately

$$F(e^+) = \frac{2}{(2\pi)^3} e^{-(E_+ + E_F^{e^-})/kT} \quad (6.5)$$

Performing the phase space integral in (6.3), using these distribution functions, gives

$$\lambda = \frac{2}{3} \frac{G_F^2}{\pi^5} \frac{(E_F^{e^-})^4 (kT)^4}{\hbar^4 c^2} e^{-E_F^{e^-}/kT}, \quad (6.6)$$

where the factors $\hbar$ and c have been re-inserted. The density of neutrinos emitted by electron positron annihilation, without Pauli inhibition, is

$$n_\nu(t) = \int_0^t dt' \lambda(t') \quad (6.7)$$

Equation (6.7) does not take into account the neutrino losses due to their escape from the core. It will be shown in the discussion of the neutrino luminosity that this is a good approximation, for the first few hundredths

of seconds after the collapse.

The Fermi energy of the electron gas is given by

$$E_F^{e-} = \hbar c (3\pi^2 n_{e-})^{1/3}, \quad (6.8)$$

n_{e-} is the number density of electrons, it is

$$n_{e-} = (\rho / m_n) \cdot Y.$$

m_n is the nucleon mass and Y is the number of electrons per nucleon. The neutrino losses during the collapse are strongly dominated by the emission of neutrinos from electron capture on protons which changes Y . It is therefore appropriate to use the value of Y at the end of the collapse, which will be shown to be about 0.35. This gives the following numerical expression for the electron Fermi energy:

$$E_F^{e-} = 2.9 \times 10^8 \frac{1}{R} \text{ MeV} \quad (6.9)$$

where R is the radius of the core expressed in cm.

The thermal history of the collapse is determined by the adiabatic condition. Let $\Gamma_1 = (d \ln p / d \ln \rho)_{ad}$ be the adiabatical index. The temperature as function of the density is then

$$T \varrho^{1-\Gamma_1} = d = \text{const.} \quad (6.10)$$

Neglecting relativistic effects, $\Gamma_1 = 4/3$ is the limit of stability for a spherically symmetric matter distribution under its own gravitational attraction. (For a discussion of possible differences when general relativistic effects are considered cf. Chandrasekhar 64). We shall therefore assume for the present discussion that $\Gamma_1 = 4/3$ during the whole collapse. The constant d is determined by choosing a point of the $T - \varrho$ diagram through which one knows that the core passes. We use the value

$$d = 2.8 \times 10^{-4} \text{ MeV cm g}^{-1/3} \quad (6.11)$$

which corresponds to $\gamma = 0.35$ and to an entropy per baryon $s \approx 2$ (cf Pethick 79). We have then the following numerical expression for the R - dependence of the temperature

$$T = 2.2 \times 10^7 \frac{1}{R} \text{ MeV} \quad (6.12)$$

when R is measured in cm. Note that both T and $E_F^{e^-}$ have the same R - dependence.

These results give the following expression for λ :

$$\lambda = \frac{2 G^2}{3 \pi^5} \left(\frac{27 \pi^2 \gamma}{16 m_n} \right)^{4/3} \cdot \frac{d^4}{R^8} \cdot \exp - \left[\frac{\hbar c}{d} \left(\frac{3 \pi^2 \gamma}{m_n} \right)^{1/3} \right], \quad (6.13)$$

or numerically

$$\lambda = \lambda_0 \frac{1}{R^8} \text{ cm}^{-3} \text{ s}^{-1}, \quad (6.14)$$

$$\lambda_0 = 1.6 \times 10^{86} \text{ cm}^5 \text{ s}^{-1}.$$

For a freely falling spherical matter distribution (of mass M), which starts from an initial state at $t = 0$ described by $R = R_0$ and $V = 0$ (i.e. the internal pressure is removed at $t = 0$), the time dependence of R is implicitly given by:

$$t = \int_{R_0}^R dr \left[\frac{2 G M}{r} - \frac{2 G M}{R_0} \right]^{1/2}. \quad (6.15)$$

Here G is the constant of gravitation. Instead of (6.15), we use the parametric representation

$$R = \frac{R_0}{2} (1 + \cos \vartheta) \quad (6.16)$$

$$t = \frac{R_0}{2} \left(\frac{R_0}{2 G M} \right)^{1/2} (\vartheta + \sin \vartheta).$$

Applying this result to the core ($M = M_c$) and inserting it in (6.7) gives

$$n_v(t) = \lambda_0 \int_0^{\vartheta(t)} \frac{1}{(1 + \cos \vartheta)^2} d\vartheta, \quad (6.17)$$

where
$$\lambda_0 = \kappa_0 \left(\frac{2}{R_0} \right)^7 \left(\frac{R_0}{2 G M_c} \right)^{1/2}.$$

$\vartheta(t)$ is given by the inverse relation to (6.16). The integral (6.17) is elementary, one finds

$$n_v(\vartheta) = \lambda_0 \cdot \sin \vartheta. \quad (6.18)$$

$$\cdot \sum_{n=0}^6 \left[\frac{6!}{(6-n)!} \cdot \frac{(12-2n-1)!!}{13!!} \frac{1}{(1 + \cos \vartheta)^{7-n}} \right]$$

$$((2x+1)!! \equiv (2x+1)(2x-1)\dots 3 \cdot 1).$$

Table 2 gives the numerical value of n_v for different times during the collapse. The initial condition is

$$\varrho(t=0) = 10^{11} \text{ g cm}^{-3}, \text{ corresponding to } R_0 = 1.7 \times 10^7 \text{ cm.}$$

We next consider the correction to the above picture when the Pauli inhibition is taken into account and we extend the calculation to times greater than the collapse time t_c . In model I, for which the core settles without bouncing, the density $\varrho = 4.8 \times 10^{14} \text{ g cm}^{-3}$ and $R = 10^6 \text{ cm}$. In model II, the model with bounce, the core collapses to

Table 2: Neutrino density and core radius for different times during the collapse. Neutrinos from electron capture on protons are omitted.

t ms	R ($\times 10^7$ cm)	$n_\nu(t)$ cm ⁻³
2.1	1.6	5.7×10^{25}
2.8	1.5	9.9 "
3.4	1.4	1.5×10^{26}
4.0	1.3	2.2 "
4.4	1.2	3.4 "
4.8	1.1	5.4 "
5.1	1.0	9.1 "
5.4	0.9	1.6×10^{27}
5.6	0.8	3.2 "
5.9	0.7	7.1 "
6.1	0.6	1.8×10^{28}
6.2	0.5	5.5 "
6.4	0.4	2.2×10^{29}
6.5	0.3	1.4×10^{30}
6.6	0.2	1.9×10^{31}
6.65	0.15	1.2×10^{32}
6.7	0.1	1.8×10^{33}

a radius $R = 10^6 \text{ cm}$ and bounces to $R = 1.5 \times 10^6 \text{ cm}$ (i.e. to $\rho = 1.4 \times 10^{14} \text{ g cm}^{-3}$) where it remains constant. The collapse time t_c is the free fall time of the core from $R = R_c$ to $R = 10^6 \text{ cm}$. It has a numerical value of $6.7 \times 10^{-3} \text{ s}$.

The treatment of neutrino transport is simple when one notes that the neutrino mean free path in the core Λ is proportional to R^3 / E_ν^2 . This means that the mean free path decreases much more rapidly than the radius. As a consequence, the neutrinos are very close to thermal equilibrium in a very short time. It is therefore a good approximation to consider the neutrino distribution function to be a Fermi Dirac function to the Fermi energy

$$E_F^\nu = (6 \pi^2 n_\nu)^{1/3} \hbar c \quad . \quad (6.19)$$

Because of the Pauli exclusion principle, no neutrinos can be produced at energies lower than E_F^ν , i.e. equation (6.3) has to be replaced by

$$\lambda = \int_{E_+ + E_- > E_F^\nu} F(e^-) \cdot F(e^+) \langle \sigma \cdot v \rangle d^3 p_+ d^3 p_- \quad (6.20)$$

which includes the effect of the Pauli suppression. For $E_+ + E_- \gg kT$ most of the energy has to come from

electrons because very few positrons have energies above kT . We therefore impose the energy condition to the electrons. The modified volume neutrino production rate is then

$$\lambda = \frac{2 G_F^2}{3 \pi^5} \frac{1}{\hbar^4 c^3} [(E_F^{e^-})^4 - (E_F^{\nu})^4] \cdot (kT)^4 e^{-E_F^{e^-}/kT}, \quad (6.21)$$

for $E_F^{\nu} < E_F^{e^-}$, otherwise λ vanishes. Equation (6.7) is then an integral equation for the neutrino density that we solve in steps. For $t < t_1$ we neglect E_F^{ν} . For $t_1 \leq t < t_2$ we approximate E_F^{ν} by $E_F^{\nu}(t_1)$, and continue this procedure to later times $t_2, t_3, \dots$, where $t_1, t_2, \dots$ is any sequence of times.

First consider $t_1 > t_c$. In this case the results of the previous discussion do not change. Table 3 gives the Fermi energies of the neutrino gas and of the electron gas for this case. The correction term in $(E_F^{\nu})^4$ in (6.21) may obviously be neglected.

We next calculate the time $t_{1/2}$ for which $E_F^{\nu} = E_F^{e^-}/2$, assuming $t_1 > t_{1/2}$. In model I the neutrino density, $n_{\frac{1}{2}}^{\nu}$, at $t_{1/2}$ is $6.4 \times 10^{36} \text{ cm}^{-3}$ and $t_{1/2} \approx n_{\frac{1}{2}}^{\nu} / \lambda$ ($R = 10^6 \text{ cm}$) = 0.04 s. In model II, $n_{\frac{1}{2}}^{\nu} = 1.9 \times 10^{36} \text{ cm}^{-3}$ and $t_{1/2} \approx n_{\frac{1}{2}}^{\nu} / \lambda$ ($R = 0.15 \times 10^7 \text{ cm}$)

Table 3: Comparison between the neutrino and the electron Fermi energies during the collapse.

t	ms	E_F^ν MeV	$E_F^{e^-}$ MeV
2.1		3.0×10^{-2}	18
4.8		6.3×10^{-2}	26
6.2		3.0×10^{-1}	58
6.7		9.5	290

= 0.3 s. Note that even at $t_{1/2}$ the correction in λ is irrelevant since

$$\frac{(E_F^{e^-})^4 - (E_F^{e^-}/2)^4}{(E_F^{e^-})^4} = 0.94, \quad (6.22)$$

and will be neglected.

The neutrino density for $t > t_c$ in the models I and II is given by

$$n_\nu(t) = \left\{ \begin{array}{l} \lambda(R = 10^6 \text{ cm}) \\ \lambda(R = 1.5 \cdot 10^6 \text{ cm}) \end{array} \right\} (t - t_c) + n_\nu(t_c). \quad (6.23)$$

Note that the positrons and the electrons remain in thermal equilibrium because the electromagnetic inter-

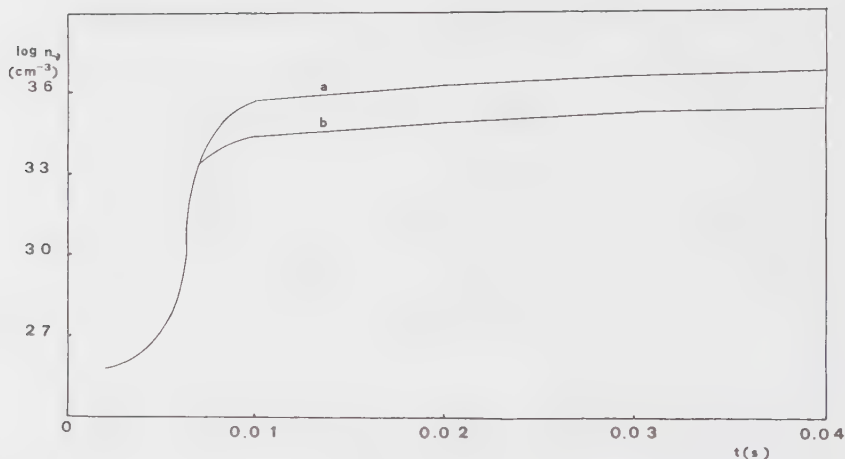


Fig.3 The logarithm of n_ν in cm^{-3} is given as function of time for the two models described in the text. Only neutrinos from electron positron pair annihilation are considered. Curve a shows n_ν for the model of collapse without bounce (model I), curve b shows n_ν for the model with bounce (model II).

actions are much faster than the electrons positrons annihilations in neutrino antineutrino pairs.

Figure 3 gives the neutrino density as a function of time for the two models described here. The neutrinos from electron capture have been omitted in Figure 3.

VII Neutrino Luminosity

In this section we discuss the neutrino luminosity of the imploding core. We consider neutrinos from the initial flash of electron capture on protons and those from electron positron pair annihilation. We do not consider electron capture on heavier nuclei. The only scattering process considered is the elastic scattering of neutrinos on ^{56}Fe nuclei. The collapse scenario is the same as in the last section, both models I and II are considered.

As we have seen, pair annihilation becomes important only when the core is already collapsed. During the collapse, the neutrino luminosity is therefore dominated by electron capture on protons. One may then discuss the two neutrino producing reactions separately.

The rate of electron capture on free protons is approximately $2 \times 10^4 \text{ s}^{-1}$ per proton for electron energies of about 24 MeV. The corresponding time is much shorter than the free fall time and allows the use of an instantaneous neutrino flash also in the context of a free collapse.

To estimate the neutrino luminosity for $t < t_c$ we use the neutrino distribution function as it is given in (4.28) with $\gamma = \gamma_{\text{max}}$. This choice takes the broadest

ning of the initial distribution function into account (although as an instantaneous effect). We do not use the assumption that the neutrinos are quasi free for very short time at the beginning of the collapse. This would be improper in the discussion of the luminosity. The choice $\gamma = \gamma_{\max}$ for $t \gg 0$ is not realistic; it assumes that the phase space available to the neutrinos around $\bar{E}_\nu$ is continuously filled. The real value of γ depends on the abundance of free protons and on the details of nuclear physics which have not been taken into account here. In the following, the energy dependence of the neutrino distribution function is assumed constant. The error made by neglecting thermalisation is of the order of 20% for the first few milliseconds. This is small enough to justify the above assumption.

The neutrino luminosity is calculated in the following way

$$L_\nu = 4\pi R^2 \int d^3p_\nu v_r(R) F_\nu(R) E_\nu, \quad (7.1)$$

where $v_r(R)$ is the radial velocity of the neutrinos at the surface of the core, $F_\nu(R)$ is the neutrino distribution function at the surface and E_ν is the energy carried by the neutrinos. The integration is over neutrino velocities directed outward from the star. As discussed

above, the neutrino distribution is that given in (4.28) and $v_r(R)$ is therefore the mean radial velocity as given in section IV. Inserting these into (7.1) gives

$$L_\nu = 4\pi R^2 \cdot 2\pi \bar{r} \cdot \gamma \cdot c \cdot 2\pi^{1/2} \Delta \bar{E}_\nu^3 \cdot \left(1 + O\left(\frac{\Delta}{\bar{E}_\nu}\right)^2\right) \quad (7.2)$$

$$= 7.6 \times 10^{30} R^4 \text{ MeV s}^{-1}$$

This result is represented graphically in figure 5 where free collapse is assumed.

The lepton concentration Y_ℓ changes in the core as neutrinos leave it. This can be estimated as follows:
The lepton concentration is given by

$$Y_\ell(t) = [N_\ell(t=0) - \int_0^t \dot{N}_\nu(t') dt'] / N_{\text{nucleons}} \quad (7.3)$$

$\dot{N}_\nu(t)$ is given by an integral of the same type as in (7.1) without the factor E_ν . Applying the same reasoning as there one gets

$$\dot{N}_\nu(t) = 4\pi R^2 \cdot 2\pi \bar{r} \cdot \gamma \cdot c \cdot \pi \cdot 2 \Delta \bar{E}_\nu^2 \quad (7.4)$$

At $t = 0$ the lepton concentration is equal to the electron concentration in ^{56}Fe , namely $Y_\ell(t=0) = 0.46$. As the collapse proceeds, the lepton concentration drops

until it reaches $Y_e \approx 0.34$. This is represented in figure 4.

The pair processes do not change Y_e in a significant way, because one produces as many neutrinos as antineutrinos. The number of electrons per nucleon denoted by Y in the previous section is somewhat less than Y_e because the density of neutrinos does not vanish. This is a small effect as one sees by comparing the neutrino density to the electron density at the end of the collapse.

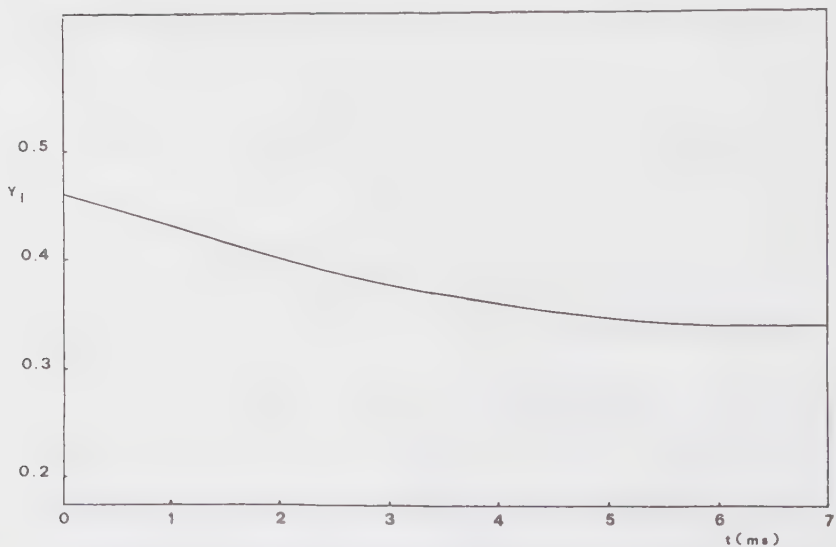


Fig.4: The lepton concentration Y_e as a function of time during the collapse.

At the end of collapse, the density of neutrinos rises steeply because of the electron positron pair annihilation. (This rise could be still steeper if the further neutronisation would be taken into account). We now calculate the contribution to the neutrino luminosity from these neutrinos in four different ways: Models I and II of the preceding section are both analysed for the cases of complete thermalisation and complete non thermalisation. The latter case is studied in order to estimate what the maximum disagreement would be for incomplete thermalisation.

The neutrino distribution function without thermalisation is artificially assumed to be $h/2 \times F(e^-)$, since no neutrinos can be produced with an energy larger than $E_F^{e^-}$. The factor h describes the amount of phase space occupied by neutrinos. It is given by the neutrino density calculated in the previous section. Since $h \leq 1/10$ during the first 0.04 s, no Pauli suppression is taken into account.

As the distribution function spans a large energy interval (in all cases), the energy dependence of the mean free path, and hence of $\langle v^r \rangle$, may not be neglected. This means that the neutrino luminosity is

$$L_\nu = 4\pi R^2 \int_0^{E_F^\nu} dE_\nu E_\nu^2 \cdot 2\pi \gamma \langle v^r \rangle(E_\nu) \cdot E_\nu \quad (7.5)$$

in the thermalised case, and

$$L_\nu = 4\pi R^2 \int_0^{E_F^e} dE_\nu E_\nu^2 \cdot 2\pi h \gamma \langle v^r \rangle(E_\nu) \cdot E_\nu \quad (7.6)$$

in the non thermalised case. The factor h is related to the density by the equation

$$h = \frac{3 n_\nu}{4\pi \gamma (E_F^e)^3} \quad (7.7)$$

The velocity $\langle v^r \rangle$ is given in section IV. Inserting explicitly the energy dependence of Λ one has

$$\langle v^r \rangle(E_\nu) = \frac{c}{(\hbar c)^2} \frac{2\pi^2 R^2 m}{G_F^2 a_0^2 A^2 E_\nu^3 M_c} \quad (7.8)$$

This together with (7.5) and (7.6) gives the following expression for the neutrino luminosity:

$$L_\nu = \frac{8\pi^4 R^4 \gamma c m (E_F^e)^2}{G_F^2 a_0^2 A^2 (\hbar c)^2 M_c} \quad (7.9)$$

in the thermalised case, and

$$L_\nu = \frac{8\pi^4 R^4 \gamma h c m (E_F^e)^2}{G_F^2 a_0^2 A^2 (\hbar c)^2 M_c} \quad (7.10)$$

in the non thermalised case. These results are represented

graphically in figure 5, together with the neutrino luminosity during the collapse.

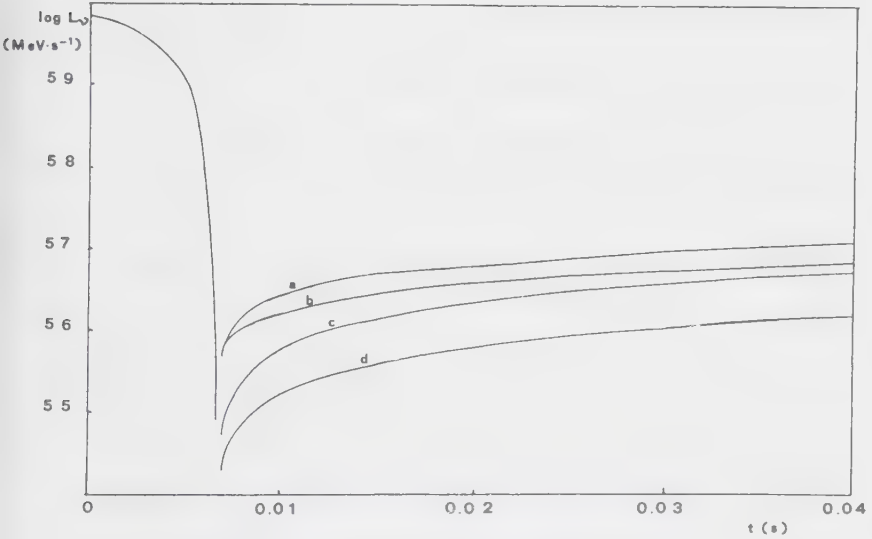


Fig. 5: The neutrino luminosity is represented as a function of time during the collapse and for a few hundredths of seconds after it. The four curves distinguish the four models of pair processes: a denotes model I (without bounce) with thermalised neutrinos, b model II (with bounce) with thermalised neutrinos, c model I with non thermalised neutrinos and d model II with non thermalised neutrinos.

It is possible to make an estimate of the pulse duration by comparing the neutrino production rate in the core, i.e. the neutrino gains, with the rate at which neutrinos leave the core, the neutrino losses. As long as the gains dominate the losses the pulse will be sustained. When this is no longer the case, the pulse decays in about one diffusion time.

For complete thermalisation, the rate at which neutrinos leave the core is given by

$$\dot{N}^{\text{losses}} = - \frac{16 \pi^4 R^4 \gamma c m E_F^2}{G_F^2 \alpha_0^2 A^2 (\hbar c)^2 M_c} , \quad (7.11)$$

by the same kind of arguments that lead to (7.4). The neutrino production rate in the whole core is

$$\dot{N}^{\text{gains}} = \frac{4\pi}{3} R^3 \lambda \quad (7.12)$$

and depends on the temperature. The temperature T_e at which the gains equal the losses is given by a comparison between (7.11) and (7.12). This gives for model I

$$1.5 \cdot 10^{57} (\kappa T_e)^4 e^{-E_F^e / \kappa T_e} = 1.7 \cdot 10^{55} \text{ s}^{-1}, \quad (7.13)$$

or numerically

$$kT_e \approx 18 \text{ MeV.} \quad (7.14)$$

As one sees from (6.12) the core temperature at $t = t_c$ is 22 MeV. Hence after a temperature drop ΔT of about 4 MeV the gains equal the losses. To estimate the time that the core needs to cool from 22 MeV to 18 MeV one computes the heat capacity c of the core and uses the thermodynamic relation $\Delta U = -L \Delta t = c \Delta T$, where U is the internal energy of the core. This gives

$$\frac{\Delta T}{\Delta t} = - \frac{L}{c} \quad , \quad (7.15)$$

The heat capacity consists of contributions from the electron gas and from the nuclei. The contributions from the positrons and the neutrinos are small because their respective density is small. The electron gas is a completely degenerate ultra relativistic gas, and therefore its heat capacity is

$$c^{e^-} = N^{e^-} \cdot \pi^2 \cdot \frac{kT}{E_F^{e^-}} \quad (7.16)$$

where N^{e^-} is the number of electrons. The nuclei form an ideal non relativistic gas with heat capacity

$$c^N = \frac{3}{2} N^N k \quad . \quad (7.17)$$

The numerical value of $c = c^{e^-} + c^n$ is

$$c \approx 3.4 \times 10^{56} \quad \text{for } kT = 22 \text{ MeV.} \quad (7.18)$$

Recall that c is a dimensionless number when the temperature and the internal energy are both expressed in MeV.

The luminosity entering (7.15) is the total luminosity of the core. Assuming that the electron-antineutrinos, the muon-neutrinos and antineutrinos give comparable contributions as the electron-neutrinos to the luminosity one has $L \approx 4 L_\nu$. A typical value for L_ν after a few hundredths of seconds is $L_\nu \approx 2 \times 10^{57} \text{ MeV} \cdot \text{s}^{-1}$ in model I with thermalisation. This gives

$$\frac{\Delta T}{\Delta t} \approx \frac{-4 L_\nu}{c} \approx -24 \text{ MeV s}^{-1}. \quad (7.19)$$

The estimated pulse duration is then

$$t_{\text{pulse}} \approx \frac{\Delta T}{\Delta T / \Delta t} \approx 0.2 \text{ s}. \quad (7.20)$$

VIII Conclusion

The general features of supernova neutrinos, which one hopes to observe soon, are the following. One expects a strong pulse of electron-neutrinos at the beginning of the collapse when the density of the inner core of the supernova is still low ($\rho \approx 10^{11} \text{ g cm}^{-3}$). These neutrinos are emitted by electron capture on protons. They lower the number of electrons per nucleon. This first pulse decays rapidly as the density rises and the mean free path of the neutrinos decreases (neutrino trapping).

The first pulse is immediately followed by a second one of less intensity in which the neutrinos are produced by thermal processes. This second neutrino pulse is expected to come together simultaneously with a pulse of electron-antineutrinos and with pulses of muon-neutrinos and antineutrinos (possibly also tau-neutrinos and antineutrinos). It is expected that the duration of the thermal pulses is of the order of a few tenths of seconds.

The momentum transfer from the neutrinos to the nuclei in the core is about one order of magnitude too small to compensate the gravitational attraction. Thus it does not produce the explosion of supernovae. The explanation of supernova explosions is probably to be found in the properties of the shock appearing when the core bounces (Arnett 80).

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References

- Arnett, W. D. 1977. Neutrino trapping during gravitational collapse of stars. *Astrophys. J.* 218: 815.
- Arnett, W. D. 1979. The physics of gravitational collapse: an overview. Talk presented at the 9th Texas Symposium on relativistic astrophysics.
- Arnett, W. D. 1980. Talk presented at the international school on Nuclear Astrophysics, 4th course: Nuclear Astrophysics, in Erice.
- Cercignani, C. 1975. Theory and application of the Boltzmann equation. Scottish Academic Press, Edinburgh and London.
- Chandrasekhar, S. 1964. Dynamical instability of gaseous masses approaching the Schwarzschild limit in general relativity. *Astrophys. J.* 140: 417.
- Epstein, R. I., Arnett, W. D. 1975. Neutronization and thermal disintegration of dense stellar matter. *Astrophys. J.* 201: 202.
- Ferziger, J. H., Kaper, H. G. 1972. Mathematical theory of transport processes in gases. North Holland Publishing Company, Amsterdam and London.
- Freedman, D. Z. 1974. Coherent effects of a weak neutral current. *Phys. Rev. D* 9: 1389.

- Freedmann, D. Z., Schramm, D. N., Tubbs, D. L. 1977. The weak neutral current and its effects in stellar collapse. Annual Reviews of Nuclear Science. Vol. 27.
- Hansen, C. J. 1968. Some weak interaction processes in highly evolved stars. Astrophysics and Space Science 1 : 499.
- Müller, E., Hillebrandt, W. 1979. A magnetohydrodynamical supernova model. Astronomy and Astrophysics 80: 147.
- Pethick, C. J. 1979. Neutrinos and stellar collapse. Talk presented at Neutrino 1979.
- Schramm, D. N., Arnett, W. D. 1975. The weak interaction and gravitational collapse. Astrophys. J. 198: 629.
- Schramm, D. N. 1979. Neutrino astronomy. Talk presented at the 9th Texas Symposium on relativistic astrophysics.
- Tubbs, D. L. 1978. Direct-simulation neutrino transport: aspects of equilibration. Astrophys. J. Supplement Series 37: 287.
- Van Riper, K.A. 1979. General relativistic hydrodynamics and the adiabatic collapse of stellar cores. Astrophys. J. 232: 558.
- Wilson, J. 1979. Stellar collapse and Supernovae. Talk presented at the 9th Texas Symposium on relativistic astrophysics.

The following works have been of general use.

Abranowitz, M., Stegun, I. A. 1972. Handbook of mathematical functions. Dover Publications Inc, New York.

Bailin, D. 1977. Weak interactions. Sussex University Press.

Gröbner, W., Hofreiter, N. 1949. Integraltafel. Springer Verlag, Wien und Innsbruck.

Landau, L., Lifchitz, E. Cours de physique théorique. Editions Mir, Moscou.

Misner, C. W., Thorne, K. S., Wheeler, J. A. Gravitation. Freeman and Company, San Francisco.

Straumann, N. 1973. Astrophysik. Course given at the University of Zurich in winter 1972-1973.

Curriculum vitae

Am 23. März 1953 wurde ich, Thierry Jean-Louis Courvoisier von Le Locle, Couvet und la Chaux-de-Fonds (Neuchâtel), in Genf geboren. In Genf besuchte ich das Gymnasium zunächst von 1968 bis 1970, dann wieder im Schuljahr 1971 bis 1972, in welchem ich die Matura (typ C) erwarb. Das Schuljahr 1970 - 1971, welches ich in Moraga (California, U.S.A.) verbrachte, schloss ich mit dem High School Diploma ab.

Vom Wintersemester 1972 - 1973 bis zum Wintersemester 1976 - 1977 studierte ich in der Abteilung IX (Mathematik und Physik) der Eidgenössischen Technischen Hochschule in Zürich. Das Studium schloss ich mit dem Diplom in Physik ab. Die Diplomarbeit habe ich in Richtung theoretische Physik gemacht.

Seit dem Frühling 1977 bin ich im Institut für Theoretische Physik der Universität als Assistent angestellt, wo meine Dissertation unter der Leitung von Herrn Prof. Dr. N. Straumann entstanden ist.

An der ETH habe ich hauptsächlich Vorlesungen bei den Professoren Baltensperger, Busch, Eckmann, Fierz, Gerber, Hunziker, Känzig, Lang, Nussbaumer, Osterwalder, Schmid, Stampfenbach, Waldmeier gehört. An der Universität habe ich Vorlesungen bei den Professoren Scharf, Straumann und Thellung gehört.

Zürich, den 7. Mai 1980

